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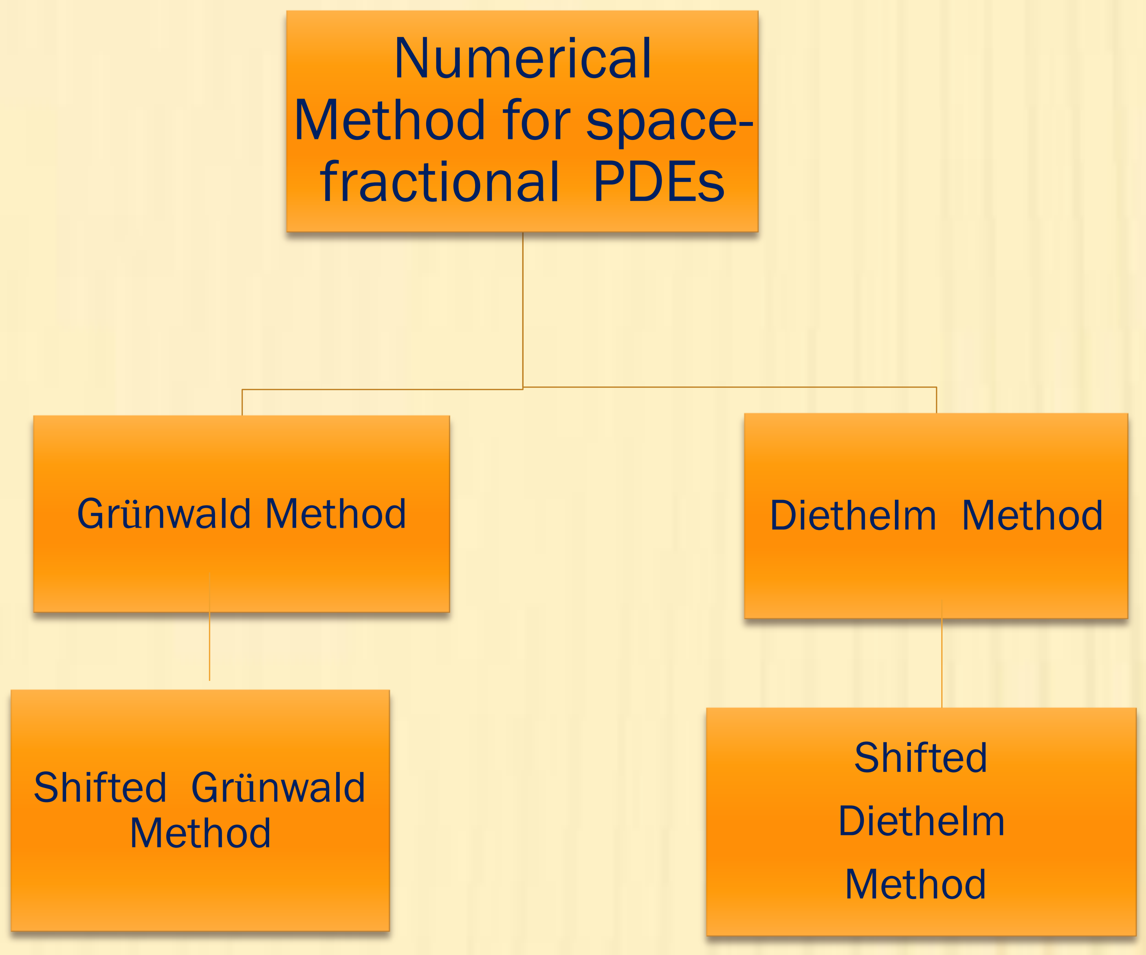
Finite difference methods for space-fractional PDEs.

1. Introduction

In the recent years many phenomena in engineering, physics, chemistry and other sciences can be described very successfully by using mathematical tools from Space-fractional PDEs such as fluid flow, finance and others. In this presentation we compare two different types of finite deference methods for solving space-fractional PDEs.

2. Methods



3. Aim

The aim of this presentation is to present two types of finite difference methods for solving space-fractional PDEs.

4. Equation

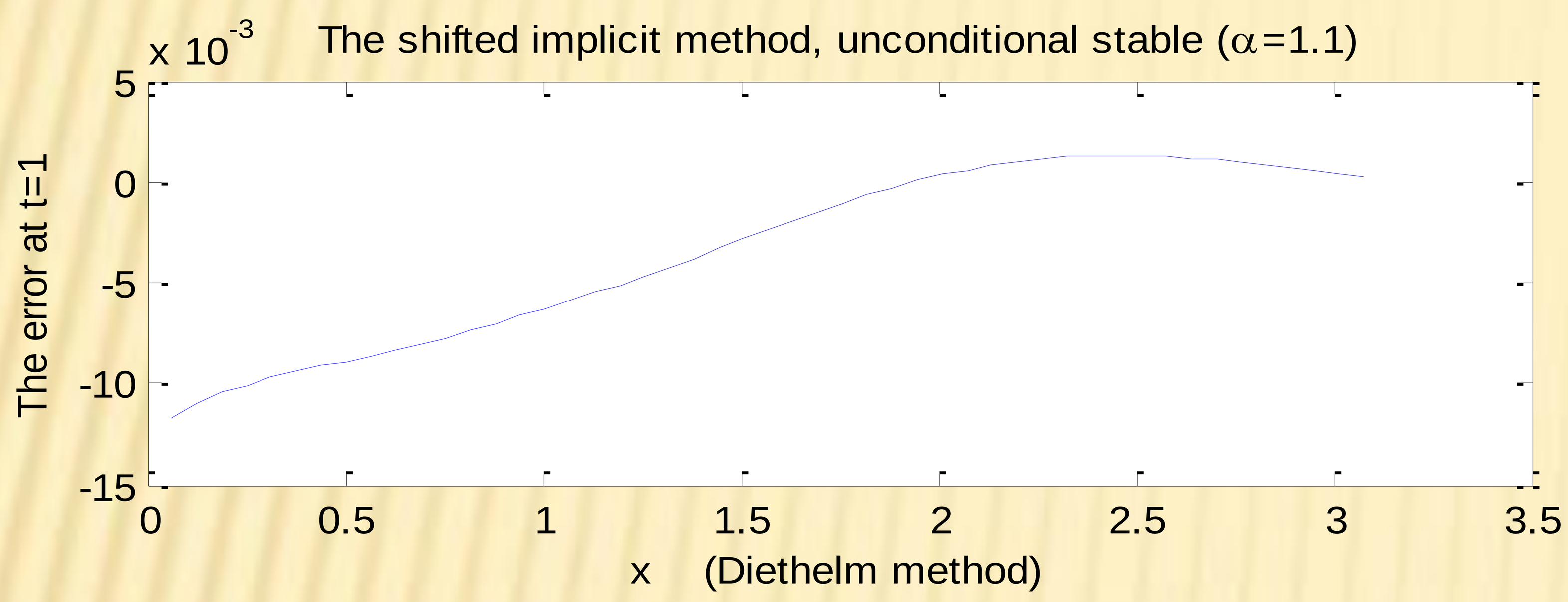
Consider the space-fractional PDEs (1<α<2)

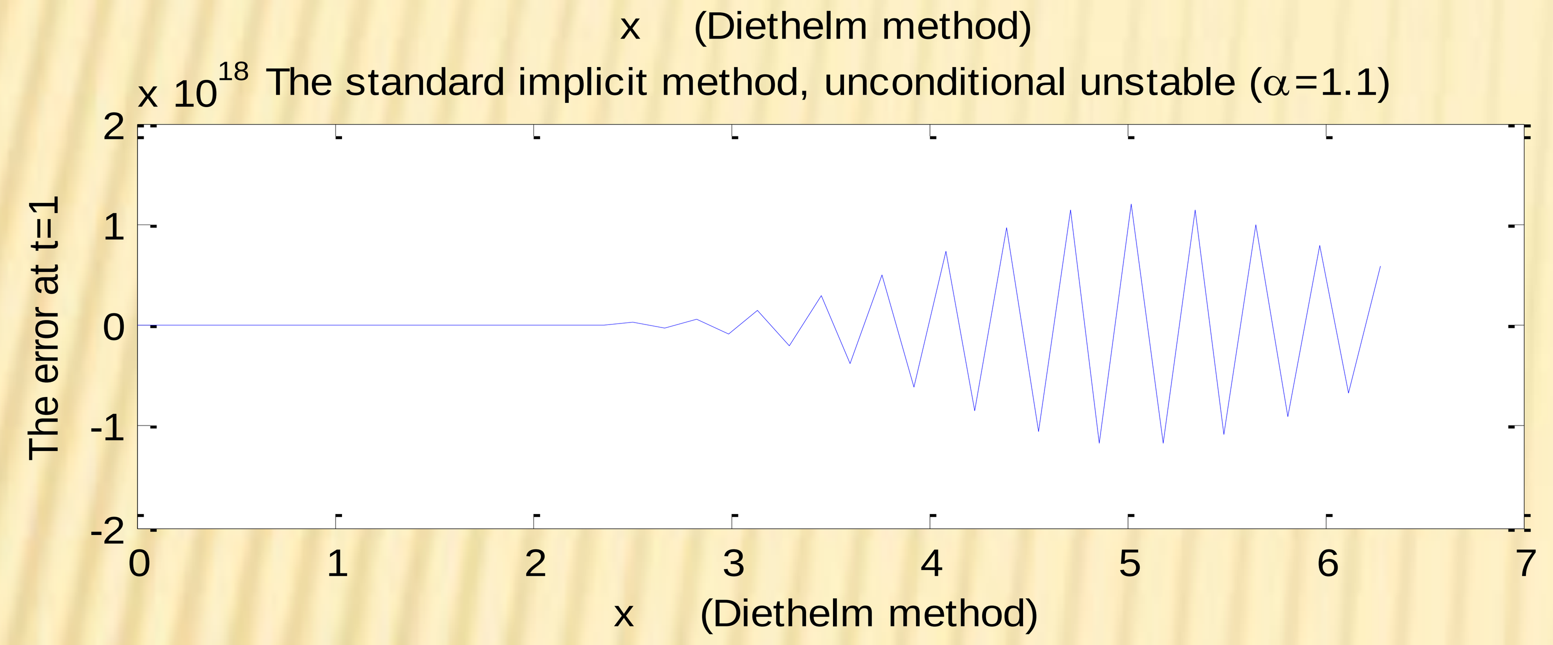
$$\frac{\partial u(x,t)}{\partial t} - {}_0^R D_x^{\alpha} u(x,t) = f(x,t) \quad 0<t<1, \quad 0<x<\pi$$

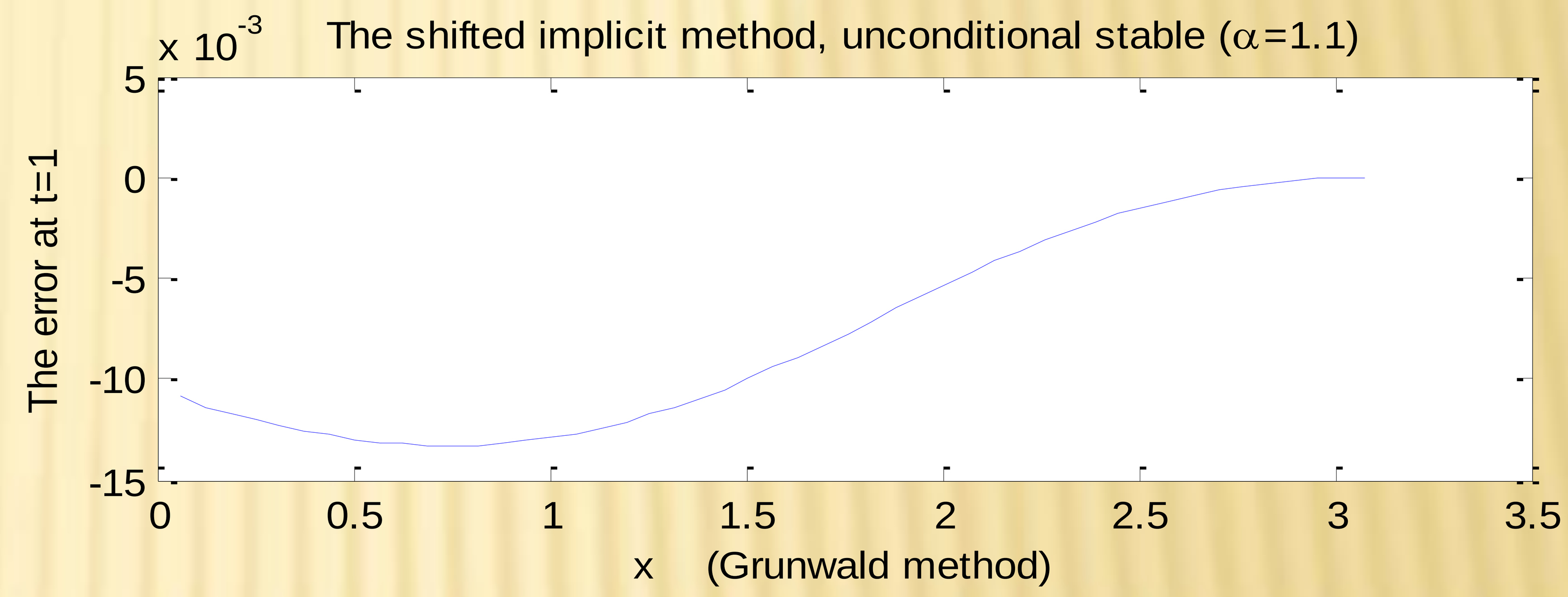
u(t,0) = u(t,1) = 0

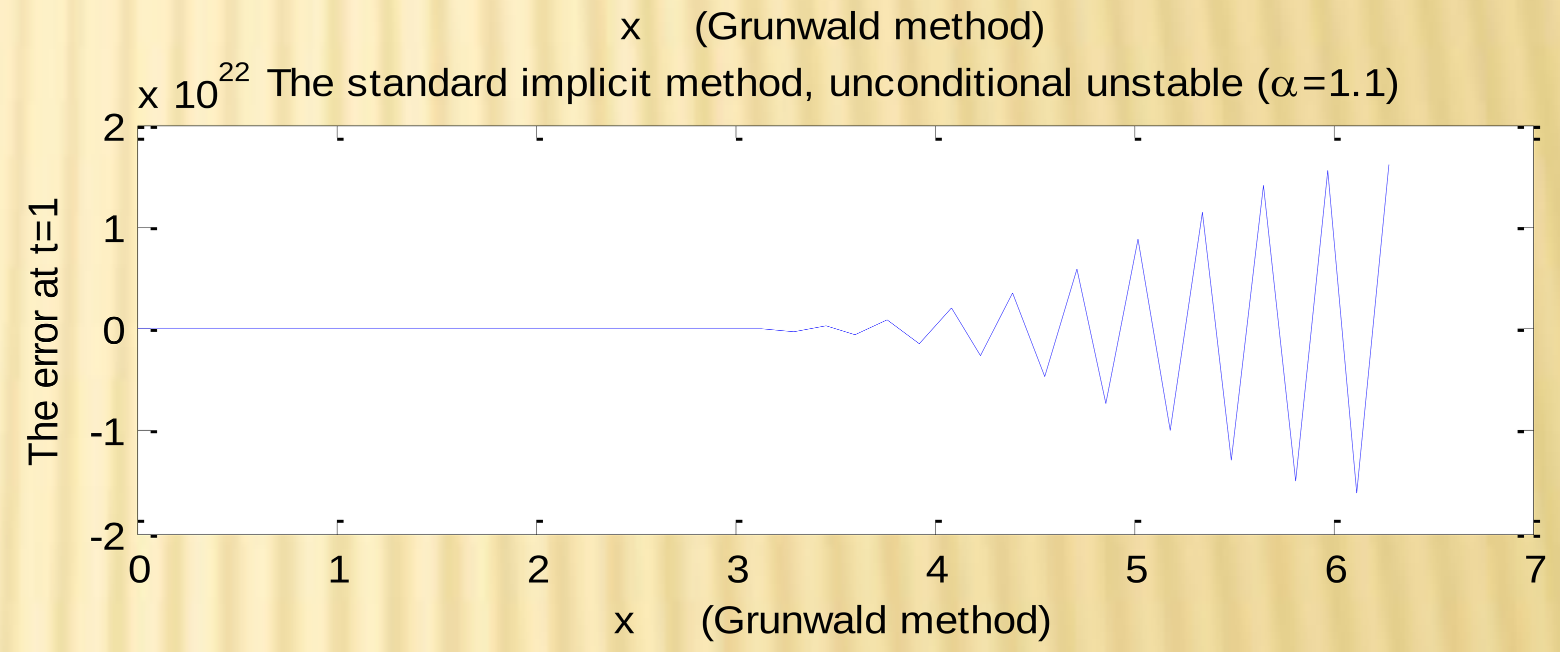
u(0,x) = u_0(x)

5.Results









Here the Diethelm's coefficient s are:

$$\Gamma(2-\alpha)w_{kj} = \begin{cases} -1, & k=0,\\ \alpha, & k=1, j=1,\\ 2-2^{(1-\alpha)}, & k=1, j>1,\\ 2k^{1-\alpha}-(k-1)^{1-\alpha}-(k+1)^{1-\alpha}, & k=2,\ldots,j-1., j\geqslant 3,\\ (\alpha-1)k^{-\alpha}-(k-1)^{1-\alpha}+k^{1-\alpha}, & k=j, j\geqslant 2,\end{cases}$$

Here the Grünwald's coefficient s are:

$$w_{kj} = (-1)^k \binom{\alpha}{k}$$

6. Result discussions

Diethelm method and Grünwald Method are both unconditionally unstable. But the shifted methods are both unconditionally stable.

7. Applications

- Random walk model
- Classical Brownian motion model
- Levy fluctuations model

8.Conclusion

The convergence order of Diethelm method is $h^{(2-\alpha)}$ and the convergence order of Grünwald method is O(h). Grünwald's method is better than Diethelm's method when 1 < α <2.

References

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Further work s

- To consider space-time-fractional PDEs
- To consider the error estimates of the different numerical methods for fractional PDEs

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